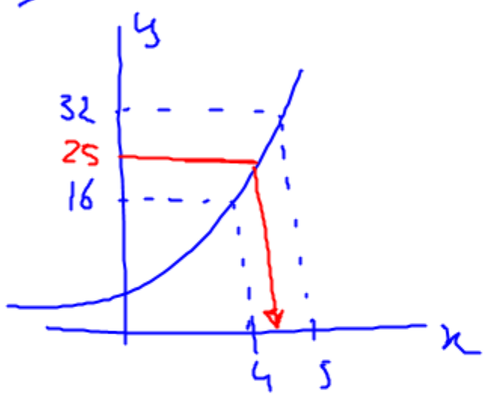
Exponentiële functies

$$y = b \cdot g^x$$

$g > 1$



Vb $y = 2^x$

$0 < g < 1$

Vb $y = \left(\frac{1}{2}\right)^x$

als $x = 5$ dan $y = 32$

als $y = 16$ dan $x = 4$

los op $2^x = 16$

$2^x = 2^4$

$x = 4$

$2^x = 32$

$2^x = 2^5$

$x = 5$

$2^x = 25$

25 is niet te schrijven als macht van 2

Dat nu toe moest je:
dat met je GRM oplossen

$y1 = 2 \wedge x$
$y2 = 25$

calc menu intersect
geeft $x =$

Vanaf dit hoofdstuk moet je $2^x = 25$ oplossen met behulp van logaritmen

$2^x = 25$

$x = {}^2\log 25$

$3^x = 50$

$x = {}^3\log 50$

V7 d $8 \cdot 4^p = \sqrt{2}$

de bedoeling om er

$g^A = g^B$ van te maken

want dan geldt

$A = B$

$2^3 \cdot (2^2)^p = 2^{\frac{1}{2}}$

$2^3 \cdot 2^{2p} = 2^{\frac{1}{2}}$

$2^{3+2p} = 2^{\frac{1}{2}}$

$3 + 2p = \frac{1}{2}$

$2p = -2\frac{1}{2}$

$p = -1\frac{1}{4}$

V1

t	0	1	2	3	4
W	25000		20000		16000

x 0,80 x 0,80

$\frac{20000}{25000} = 0,80$ $\frac{16000}{20000} = 0,80$

$g_{2 \text{ jaar}} = 0,80$

$g_{1 \text{ jaar}} = 0,80 = 0,9$

$\frac{1}{2}$

V3 $N = b \cdot g^t$

$x^p \cdot x^q = x^{p+q}$

d) $N = 457 \cdot 2^{t+4}$

$N = 457 \cdot 2^t \cdot 2^4$

$N = 457 \cdot 2^4 \cdot 2^t$

$N = 7312 \cdot 2^t$

$b = 7312$

$g = 2$

e $N = 25 \cdot 0{,}5^{-2t+1}$

$N = 25 \cdot 0{,}5^{-2t} \cdot 0{,}5^{1}$

$N = 25 \cdot 0{,}5^{1} \cdot 0{,}5^{-2t}$

$N = 12{,}5 \cdot 0{,}5^{-2t}$

$N = 12{,}5 \cdot \left(0{,}5^{-2}\right)^{t}$

$N = 12{,}5 \cdot \left(\frac{1}{0{,}5^{2}}\right)^{t}$

$N = 12{,}5 \cdot \left(\frac{1}{0{,}25}\right)^{t}$

$N = 12{,}5 \cdot 4^{t}$

$(x^{p})^{q} = x^{p \cdot q}$

$x^{-p} = \frac{1}{x^{p}}$

$b = 12{,}5$

$g = 4$

V_7 e $6 \cdot \sqrt{6}^{x} = \frac{1}{6}$

$q^A = q^B$

$A = B$

$6^1 \cdot 6^{\frac{1}{2}x} = 6^{-1}$

$6^{1+\frac{1}{2}x} = 6^{-1}$

$1 + \frac{1}{2}x = -1$

$\frac{1}{2}x = -2$

$x = -4$

f

$$5^{2x} \cdot 5^{3x+3} = 1$$

$$5^{2x} \cdot 5^{3x+3} = 5^0$$

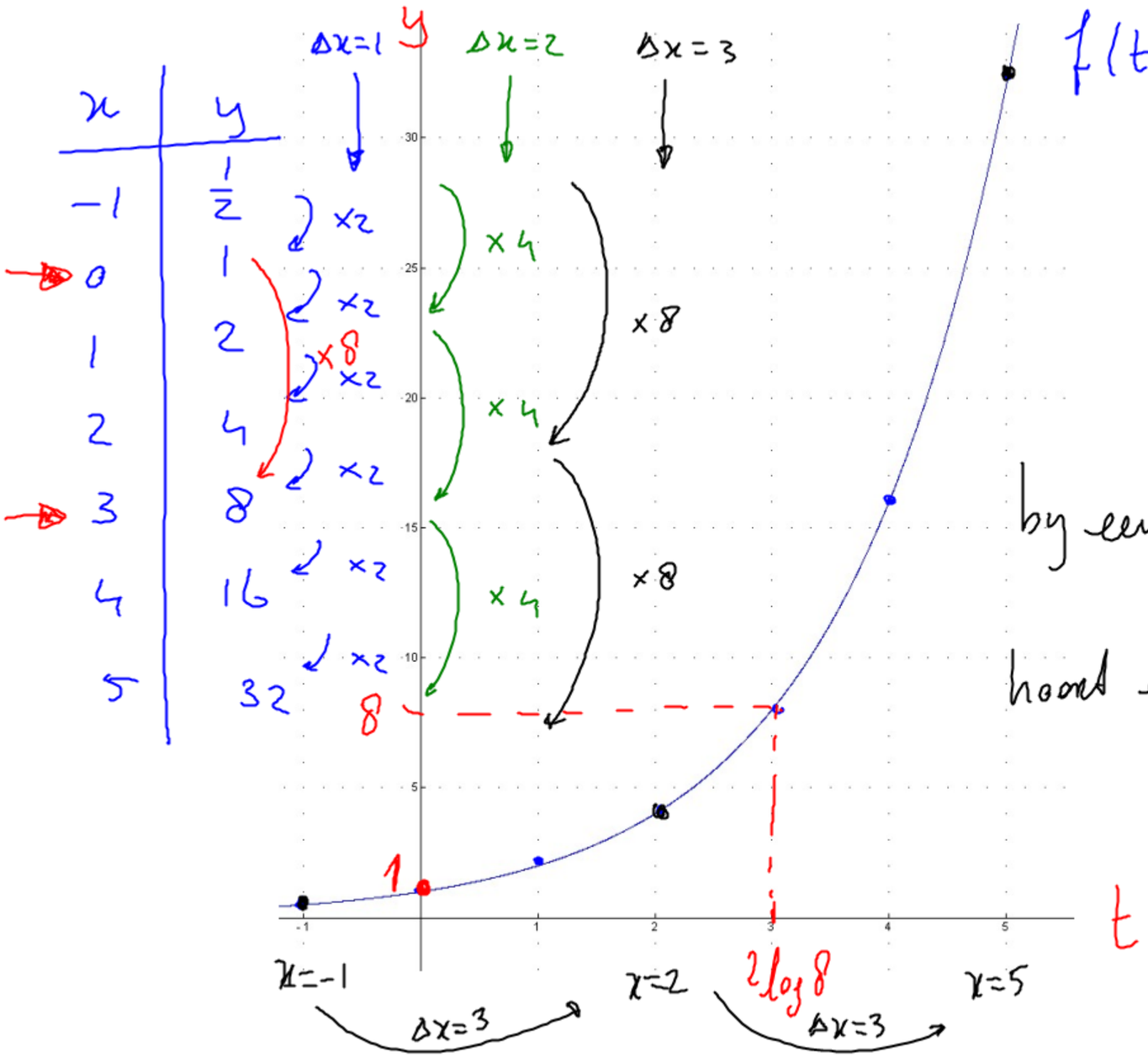
$$5^{2x+3x+3} = 5^0$$

$$5^{5x+3} = 5^0$$

$$5x+3 = 0$$

$$5x = -3$$

$$x = \frac{-3}{5}$$



$f(t) = 2^t$

$y = 2^t$

bij een groeitijd van 3

hoort een vermenigvuldiging van $2^3 = 8$

$^2\log 8$ de tijd die nodig is om bij exponentiële groei met groeifactor 2 de hoeveelheid 8 keer zo groot te laten worden

We praten hier over een zgn. groeitijd

$^2\log 8$ is de oplossing van $2^k = 8$

$^{2}\log 8 = 3$ want $2^{3} = 8$

Vb $^{2}\log x = 5$ $\quad 2^{5} = x$

$32 = x$

Vb $^{2}\log 16 = x$ $\quad 2^{x} = 16$

$2^{x} = 2^{4}$

$x = 4$

1 formule

$$A(t) = 10 \cdot 1{,}5^{t}$$

$$A(t) = 20$$

los op: $10 \cdot 1{,}5^{t} = 20 \quad (:10)$

$1{,}5^{t} = 2$

$y1 = 10 \cdot 1{,}5 \wedge x$

$y2 = 20$

intersect

$t = {}^{1{,}5}\log 2$

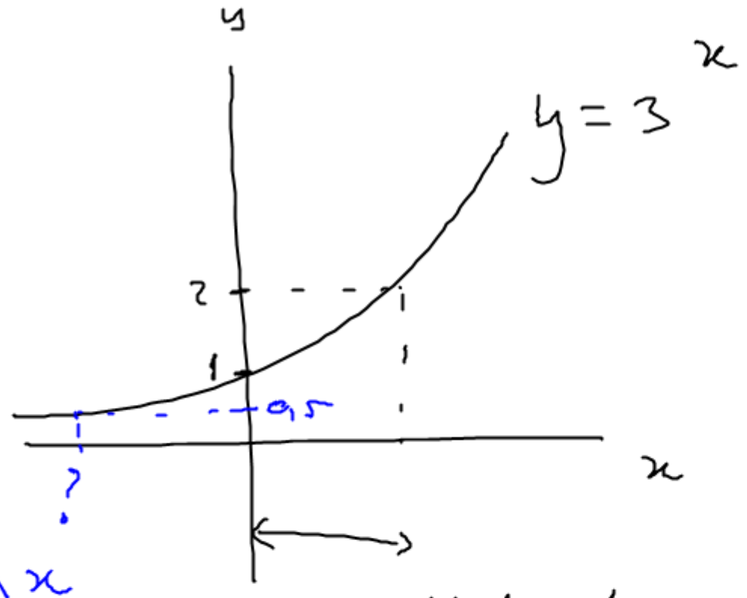
exact

$y1 = 1{,}5 \wedge x$

$y2 = 2$

intersect

5



verdubbelingstijd

$y_1 = 3 \wedge x$
$y_2 = 0,5$
window, schets
Intersect geeft $x = -0,6$ ✗

bereken je met $1 \cdot 3^x = 2$

$3^x = 0,5$
$x = {}^3\log 0,5$

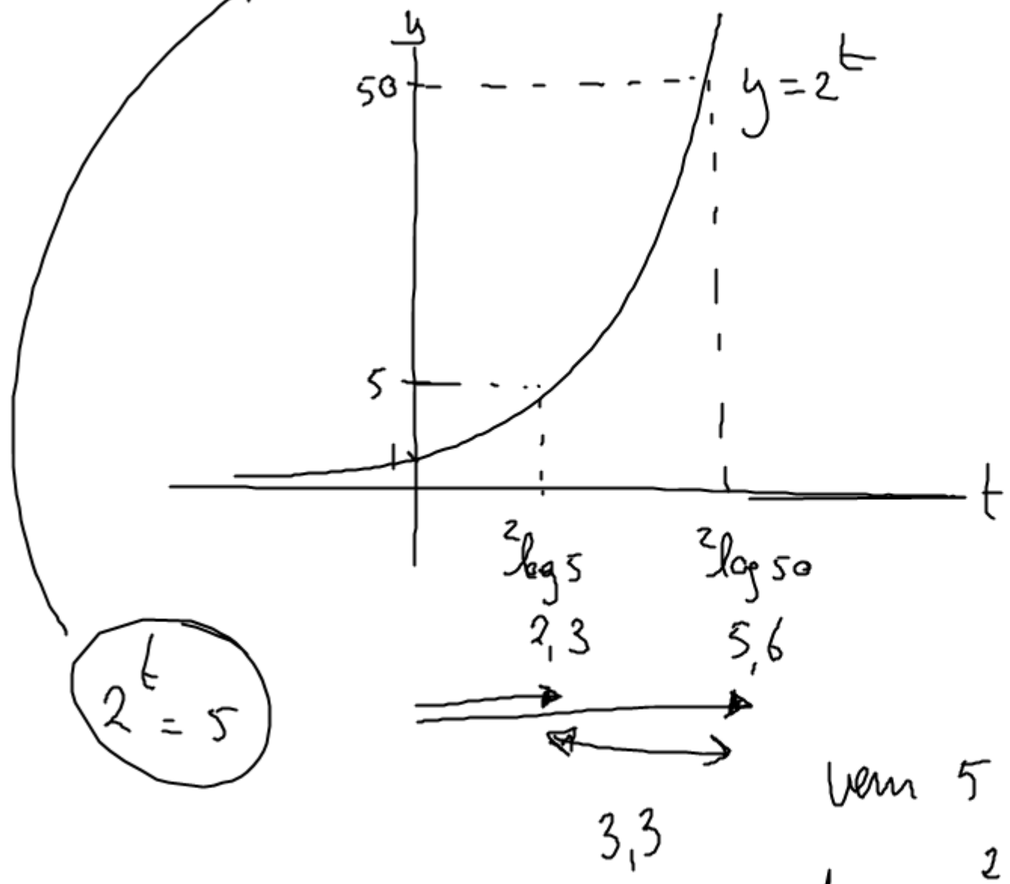
$x = {}^3\log 2$

dus $^3\log 2$ is de oplossing van $3^x = 2$

8

$2^t = 5$ $\qquad$ $2^t = 50$

$t = {}^2\log 5$ $\qquad$ $t = {}^2\log 50$



Vem 5 naar 50 is 10 keer zo groot

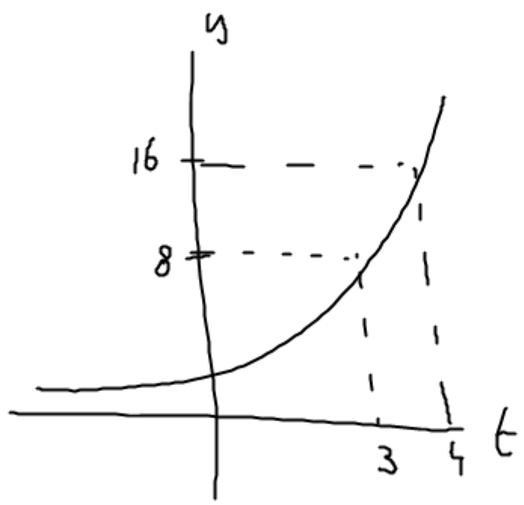
dus ${}^2\log 10 = 3,3$

10

de $^2\log \ldots$ heeft te maken met $2^{\ldots}$

$^2\log 8 = 3$ want $2^3 = 8$

$^2\log 16 = 4$ want $2^4 = 16$



11 (5) $^{5}\log 30$

exponentiële grafiek met grondgetal 5 !

$y = 1.5^{t}$

t	0	1	2	3
y	1	5	25	125

hier tussen in ligt $^{5}\log 30$

dus $^{5}\log 30 = 2,\ldots$

18 $^{4}\log 20 = \ldots^{?}$ hoe groot is dit getal ?

Om deze vraag te beantwoorden doen we eerst iets anders

$10^{x} = 4$

$x = {}^{10}\log 4$

$x = \log 4$

dus $4 = 10^{\log 4}$ en

$10^{x} = 20$

$x = {}^{10}\log 20$

$x = \log 20$

$20 = 10^{\log 20}$

deze twee resultaten zullen we in us de ~~bovenste~~ bovenste regel

$^4\log 20$ is de oplossing van

$4^t = 20$ dus $t = {}^4\log 20$

dus $\left(10^{\log 4}\right)^t = 10^{\log 20}$

$10^{t \cdot \log 4} = 10^{\log 20}$

$t \cdot \log 4 = \log 20$

$$t = \frac{\log 20}{\log 4}$$

$(x^p)^q = x^{p \cdot q}$

$g^A = g^B$ dus $A = B$

Dus $^4\log 20 = \dfrac{\log 20}{\log 4}$

huiswerk dinsdag na Pasen 26/4

Hm opg 22

Vb

$4^{t} = 6$

$t = {}^{4}\log 6$

$\left(t = \frac{\log 6}{\log 4} = \dots \right)$

20 c)

$4^{2x} = 6$

$2x = {}^{4}\log 6$

$x = \frac{{}^{4}\log 6}{2}$

22 $P = 100 \cdot 0{,}96^{t}$

d maak de 't' vrij

$t =$ iets met P

$\frac{P}{100} = 0{,}96^{t}$

$t = {}^{0{,}96}\log \frac{P}{100}$

$^{2}\log 8 = 3$ want $2^{3} = 8$

$8 = 2^{3}$

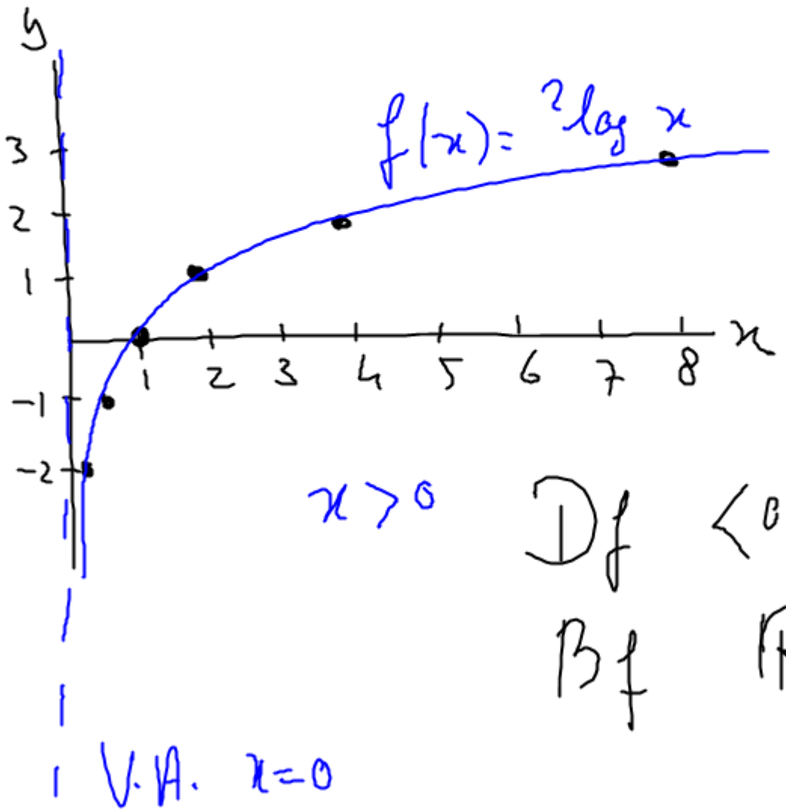
$3 = {}^{2}\log 8$

$t = \frac{\log \frac{P}{100}}{\log 0{,}96}$

25 $f(x) = {}^2\log x$

$h(x) = 2^x$

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
y	-2	-1	0	1	2	3



$x > 0$

$D_f \quad \langle 0, \to \rangle$

$B_f \quad \mathbb{R}$

de opl van $2^x = 8$

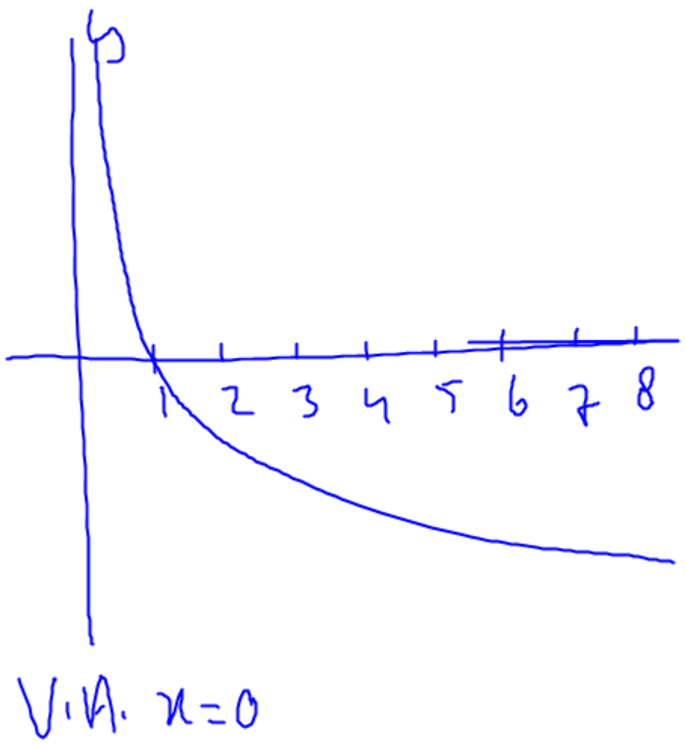
$x = 3$

de opl van $2^x = 0$

Ø

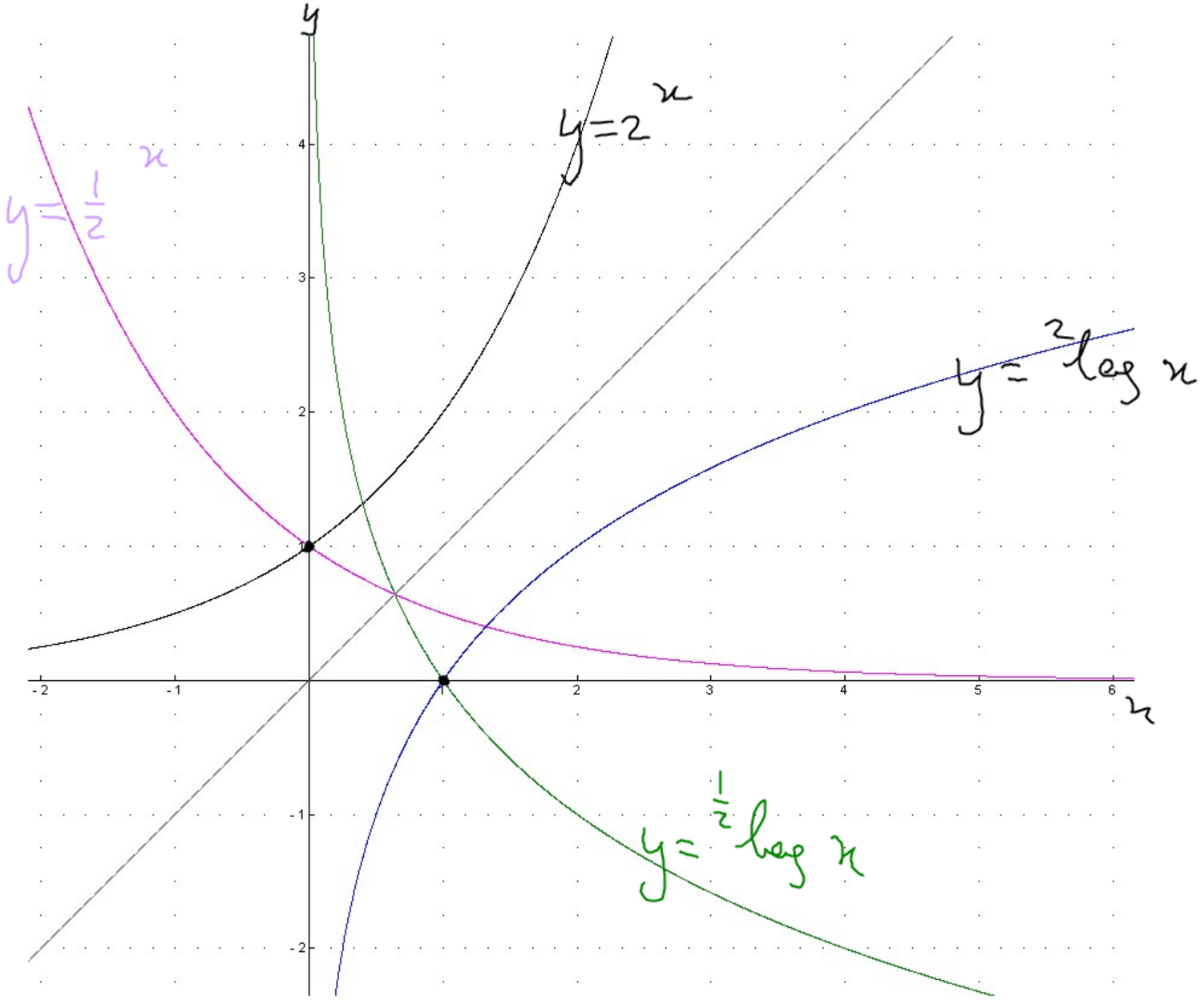
$g(x) = \log_{\frac{1}{2}} x$

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
y	3	2	1	0	−1	−2	−3



D_g $\langle 0, \to \rangle$

B_g $\mathbb{R}$


y
x
$y=2^x$
$y=\frac{1}{2}^x$
$y=^2\log x$
$y=^{\frac{1}{2}}\log x$
-2
-1
1
2
3
4
5
6
-2
-1
1
2
3
4

Woensday 1e uur

huiswerk thn opgave 27

24 a $g_{30\,jaar} = \frac{1}{2}$

$g_{1\,jaar} = \left(\frac{1}{2}\right)^{\frac{1}{30}} = 0{,}977$

$P = 100 \cdot 0{,}977^{t}$ benaderingswaarde

$P = 100 \cdot \left(\left(\frac{1}{2}\right)^{\frac{1}{30}}\right)^{t}$

$P = 100 \cdot \left(\frac{1}{2}\right)^{\frac{1}{30} \cdot t}$ } exact

$^2\log 8 = 3$ want $2^3 = 8$

b $\quad 100 \cdot \left(\frac{1}{2}\right)^{\frac{1}{30}t} = 75$

$\left(\frac{1}{2}\right)^{\frac{1}{30}t} = 0{,}75$

$\frac{1}{30}t = {}^{\frac{1}{2}}\log 0{,}75 \qquad \times 30$

$t = 30 \cdot {}^{\frac{1}{2}}\log 0{,}75$

precies

$t = 12{,}4511$ jaar

12 jaar en $0{,}4511 \times 12 =$ 5,4 mnd

$100 \cdot 0{,}977^t = 75$

$0{,}977^t = 0{,}75$

$t = {}^{0{,}977}\log 0{,}75$

12 jaar en 5 mnd

afgerond

c

0,1 % van de beginwaarde (100 %) is 0,1
0,2 % 0,2

$100 \cdot \left(\frac{1}{2}\right)^{\frac{1}{30}t} = 0{,}1$

$t = 30 \cdot {}^{\frac{1}{2}}\log 0{,}001$

$t = 299$

$100 \cdot \left(\frac{1}{2}\right)^{\frac{1}{30}t} = 0{,}2$

$t = 30 \cdot {}^{\frac{1}{2}}\log 0{,}002$

$t = 267$

2g $f(x) = {}^2\log(x+3)$

Domein $x+3>0$ dus $x>-3$ $\langle -3, \to \rangle$

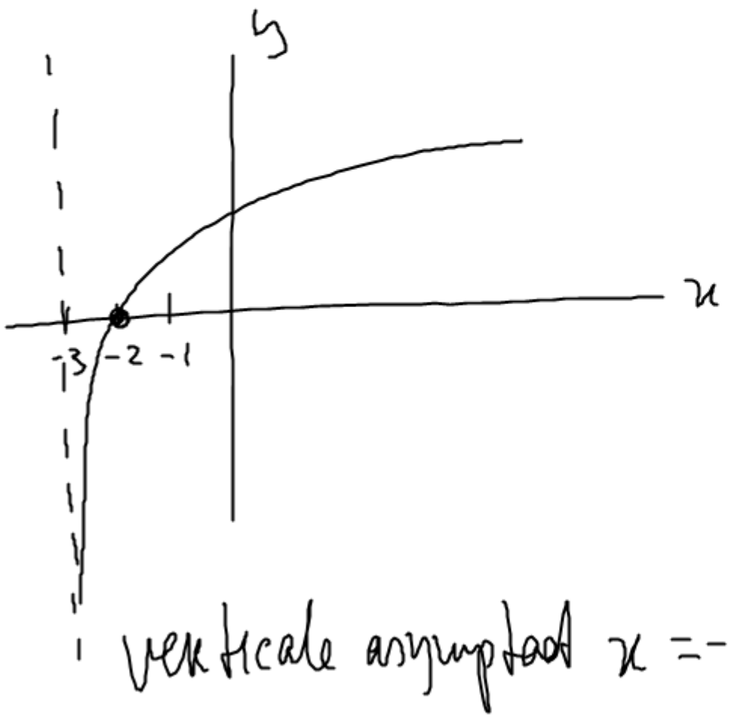
Bereik $\mathbb{R}$

Nulpunt los op: ${}^2\log(x+3) = 0$

$x+3 = 2^0$

$x+3 = 1$

$x = -2$ $(-2, 0)$



Rekenregels

$$^2\log 4 + {}^2\log 8 = {}^2\log 32$$

Lijd om de tak $4 \times$ zo groot te maken + Lijd om de tak $8 \times$ zo groot te maken = 5 "uur" in deze lijd wendt de tak $4 \times 8 = 32$ keer zo groot

$$^2\log 32 =$$

$$^2\log 4 \cdot 8 =$$

$$^2\log 4 + {}^2\log 8$$

$$\longrightarrow \quad {}^2\log a \cdot b = {}^2\log a + {}^2\log b$$

$$\longrightarrow \quad {}^2\log \frac{a}{b} = {}^2\log a - {}^2\log b$$

$$\longrightarrow \quad a \cdot {}^2\log b = {}^2\log b^a$$

Dinsdag 10 mei 6e uur

huiswerk t/m § 7.5

Test hfdst 7

maandag 23 mei ipv ma 16 mei

34 f

$$3 \cdot {}^{0,5}\log p - {}^{0,5}\log p^2$$

$${}^{0,5}\log p^3 - {}^{0,5}\log p^2$$

$${}^{0,5}\log \frac{p^3}{p^2}$$

$${}^{0,5}\log p$$

36 $f(t) = \log_{10} t$

$g(t) = \log_{10} 10t$

$h(t) = \log_{10} 0{,}01t$

a $g(t) = \log 10.t$

$= \log 10 + \log t$

$= 1 + \log t$

b $h(t) = \log 0{,}01.t$

$= \log 0{,}01 + \log t$

$= \log 10^{-2} + \log t$

$= -2 + \log t$

c

$$4 + \log p$$

$$4 \cdot 1 + \log p$$

$$4 \cdot \log 10 + \log p$$

$$\log 10^4 + \log p$$

$$\log 10^4 \cdot p$$

$$\log 10000\, p$$

k

$$6 + {}^2\log q$$

$$6 \cdot 1 + {}^2\log q$$

$$6 \cdot {}^2\log 2 + {}^2\log q$$

$${}^2\log 2^6 + {}^2\log q$$

$${}^2\log 64 + {}^2\log q$$

$${}^2\log 64q$$

37 DAB $D = 35{,}9 \log v + 4{,}1$

ZOAB $D = 28{,}1 \log v + 16{,}0$

a $75 = 35{,}9 \log v + 4{,}1$

$70{,}9 = 35{,}9 \log v$

$\frac{70{,}9}{35{,}9} = \log v$

$^{10}\log v = 1{,}97$

$v = 10^{1{,}97}$

$^{2}\log 8 = 3$ want $2^{3} = 8$

b $D = 35{,}9 \log v + 4{,}1$

$D = 35{,}9 \log 2 \cdot v + 4{,}1$

$D = 35{,}9 (\log 2 + \log v) + 4{,}1$

$D = \underline{\underline{35{,}9 \cdot \log 2}} + 35{,}9 \cdot \log v + 4{,}1$

dat stuk kond ek by

$35{,}9 \cdot \log 2 \simeq 11$

C $\quad ZOAB < DAB$

$$28{,}1 \log v + 16{,}0 < 35{,}9 \log v + 4{,}1$$

$$28{,}1 \log v + 16{,}0 = 35{,}9 \log v + 4{,}1$$

$$28{,}1 \log v + 11{,}9 = 35{,}9 \log v$$

$$11{,}9 = 7{,}8 \log v$$

$$\log v = \frac{11{,}9}{7{,}8}$$

$$\log v = 1{,}5$$

$$v = 10^{1{,}5}$$

Rechts

$v > 10^{1{,}5}$

d ZOAB + 4 < DAB

ZOAB + 4 = DAB

ent zie c

41 e

$6 \cdot 2^{3a} = 800$

$2^{3a} = 133\frac{1}{3}$

$3a = {}^{2}\log 133\frac{1}{3}$

$a = \dfrac{{}^{2}\log 133\frac{1}{3}}{3}$

$2^{3a} = 133\frac{1}{3}$

$\left(2^{3}\right)^{a} = 133\frac{1}{3}$

$8^{a} = 133\frac{1}{3}$

$a = {}^{8}\log 133\frac{1}{3}$

42 e $\quad 1 - \boxed{{}^{4}\log(x+12)} = 0$

${}^{4}\log(x+12) = 1$

$x + 12 = 4$

$x = -8$

43 b

$$^2\log(2x+1) + {}^2\log 7 = 4$$

$$^2\log((2x+1)\cdot 7) = 4$$

$$(2x+1)\cdot 7 = 2^4$$

$^2\log p = 4$

$p = 2^4$

$$(2x+1)7 = 16$$

$$2x+1 = \frac{16}{7}$$

$$2x = \frac{16}{7} - 1$$

$$2x = \frac{9}{7}$$

$$x = \frac{\frac{9}{7}}{2} = \frac{9}{14}$$

$y1 = -2 + 3\log(x)$

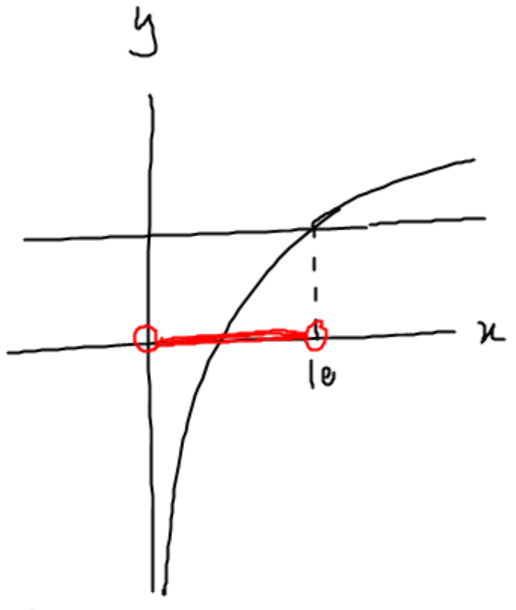
45 e $\quad -2 + 3 \cdot \log(x) < 1$

$-2 + 3 \cdot \log(x) = 1$

$3\log(x) = 3$

$\log(x) = 1$

$x = 10$



$0 < x < 10$

$\langle 0, 10 \rangle$

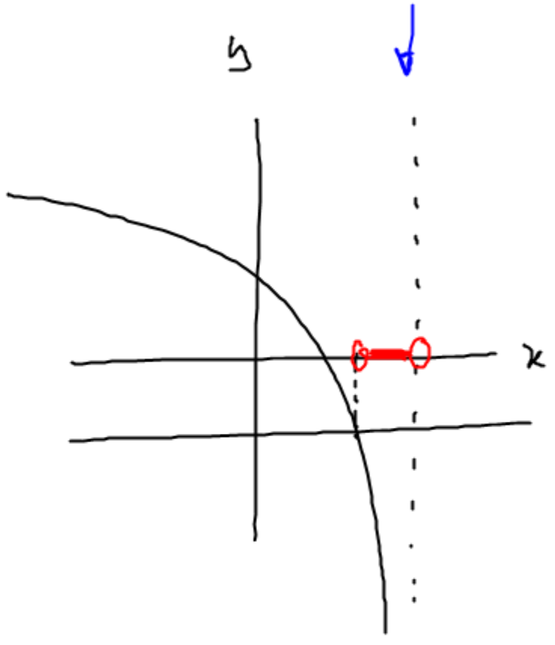
Vanwege het domein

4 r b $\quad {}^2\log(4-2x) < -1$

$4-2x=0$

$2x=4$

$x=2$ VA



${}^2\log(4-2x) = -1$

$4-2x = 2^{-1}$

$4-2x = \frac{1}{2}$

$-2x = -3\frac{1}{2}$

$x = \frac{-3\frac{1}{2}}{-2}$

$x = \frac{7}{4}$

Meandag
Luisweek
Hw opgave 4g

$\frac{7}{4} < x < 2$

vanwege het domein

47 $f(x) = 2 - {}^3\log(2x+4)$

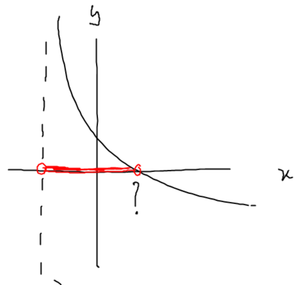
Domein: $2x+4>0$

$2x>-4$

$x>-2$

Bereik: alles $\mathbb{R}$

VA: $x=-2$



$2 - {}^3\log(2x+4) = 0$

${}^3\log(2x+4) = 2$

$2x+4 = 3^2$

$2x+4 = 9$

$2x = 5$

$x = 2\frac{1}{2}$

S $f(x)>0$

$-2 < x < 2\frac{1}{2}$

$\langle -2, 2\frac{1}{2} \rangle$

48 $C = 1{,}6 \cdot 10^{0{,}4(t-30)}$

b $C(30) = 1{,}6 \cdot 10^{0{,}4(30-30)} = 1{,}6 \cdot 10^{0} = 1{,}6$

$C(35) = 1{,}6 \cdot 10^{0{,}4(35-30)} = 1{,}6 \cdot 10^{2} = 160$

$C(40) = 1{,}6 \cdot 10^{0{,}4(40-30)} = 1{,}6 \cdot 10^{4} = 16000$

c $500 = 1{,}6 \cdot 10^{0{,}4(t-30)}$

$10^{0{,}4(t-30)} = \frac{500}{1{,}6} = 312{,}5$

$0{,}4(t-30) = \log 312{,}5$

$t-30 = 2{,}5 \cdot \log 312{,}5$

$t = 30 + 2{,}5 \log 12.5 =$

50 a

hoek	360	:40	0,9
omtrek	40000	:40	100

$$R = \log 0{,}001 + 1{,}66 \log 0{,}9 + 3{,}08$$

$$R = -3 + -0{,}076 + 3{,}08 = 0$$

b

$\log 0{,}04$

-2

als $\log$ hoger is dan R plus 1

b $R = \log u + 1{,}66 \log D + 3{,}08$

$R = \log_{10} u + 1{,}66 \log D + 3{,}08$

$R = \log_{10} + \log u + 1{,}66 \log D + 3{,}08$

$R = 1 + \frac{}{}$

c $7{,}1 = \log u + 1{,}66 \log 55 + 3{,}08$

$4{,}02 = \log u + 2{,}89$

$1{,}13 = \log u$

$u = 10^{1{,}13} = 13{,}5 \text{ mm}$

d

$$7,1 = \log U + 1,66 \log D + 3,08$$

$$\log U + 1,66 \log D = 4,02 \quad \text{log wegwerken}$$

$$\log U + \log D^{1,66} = 4,02$$

$$\log U \cdot D^{1,66} = 4,02$$

$$U \cdot D^{1,66} = 10^{4,02} \quad \text{D vrijmaken}$$

$$D^{1,66} = \frac{1}{U} \cdot 10^{4,02}$$

$$D = \left(\frac{1}{U} \cdot 10^{4,02}\right)^{\frac{1}{1,66}}$$

$$D = \left(U^{-1} \cdot 10^{4,02}\right)^{\frac{1}{1,66}}$$

$$D = \left(U^{-1}\right)^{\frac{1}{1,66}} \cdot \left(10^{4,02}\right)^{\frac{1}{1,66}}$$

$$D = U^{-\frac{1}{1,66}} \cdot 10^{\frac{4,02}{1,66}}$$

$$D = p \cdot U^{q}$$

51 a

$$16 \cdot 2^{t-6} = \sqrt{2}$$

$$2^4 \cdot 2^{t-6} = 2^{\frac{1}{2}}$$

$$2^{4+t-6} = 2^{\frac{1}{2}}$$

$$2^{t-2} = 2^{\frac{1}{2}}$$

$$t-2 = \frac{1}{2}$$

$$t = 2\frac{1}{2}$$

51 e

$$2 \cdot {}^{5}\log 3 + {}^{5}\log(6x-4) = 2$$

$${}^{5}\log 3^{2} + {}^{5}\log(6x-4) = 2$$

$${}^{5}\log 9 + {}^{5}\log(6x-4) = 2$$

$${}^{5}\log\left(9 \cdot (6x-4)\right) = 2$$

$$9 \cdot (6x-4) = 5^{2}$$

$$54x - 36 = 25$$

$$54x = 61$$

$$x = \frac{61}{54}$$

T1 A $1,07$

B $1,07^{\frac{1}{12}} = 1,0057$

C $\frac{+}{-} \frac{\pm 0}{7}$

$T = 10$

$1,07^{x} = 2$

$1,07 \log 2 = x$

$x = 10$ jaar